# APPENDIX B

THE NEWTON-RAPHSON ITERATION TECHNIQUE APPLIED TO THE COLEBROOK EQUATION

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## THE NEWTON-RAPHSON ITERATION TECHNIQUE

Since the value for $f$ in the Colebrook equation cannot be explicitly extracted from the equation, a numerical method is required to find the solution. Like all numerical methods, we first assume a value for $f$, and then, in successive calculations, bring the original assumption closer to the true value. Depending on the technique used, this can be a long or slow process. The Newton-Raphson method has the advantage of converging very rapidly to a precise solution. Normally only two or three iterations are required.

The Colebrook equation is:

$$\frac{1}{\sqrt{f}} = -2\ \log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e\,\sqrt{f}}\right)$$

The technique can be summarized as follows:

1. Re-write the Colebrook equation as:

$$F = \frac{1}{\sqrt{f}} + 2\ \log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e\,\sqrt{f}}\right) = 0$$

2. Take the derivative of the function $F$ with respect to $f$:

$$\frac{dF}{df} = -\frac{1}{2} f^{-3/2}\left(1 + \frac{2.51\ \log_{10} e}{\left(\frac{\varepsilon}{3.7\,D} + \frac{2.52\ f^{-1/2}}{\mathrm{Re}}\right)\mathrm{Re}}\right)$$

3. Give a trial value to $f$. The function $F$ will have a residue (a non-zero value). This residue (RES) will tend towards zero very rapidly if we use the derivative of $F$ in the calculation of the residue.

$$f_n = f_{n-1} - RES \quad with \quad RES = \frac{F}{\frac{dF}{df}}$$

For $n = 0$ assume a value for $f_0$, calculate $RES$ and then $f_1$, repeat the process until $RES$ is sufficiently small (for example $RES < 1 \times 10^{-6}$ ).