THE APPLICATION OF THERMODYNAMICS TO PUMP SYSTEMS

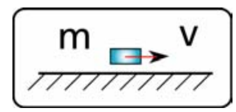
2

2.0 ENERGY AND THERMODYNAMIC PROPERTIES

This chapter requires some introduction to thermodynamic properties and states. No need to panic, we will use only what we need to know to calculate Total Head. Several measurable quantities are used to define the state of a substance: temperature (T), pressure (p), velocity (v), and elevation (z). A specific type of energy corresponds to each of these quantities.

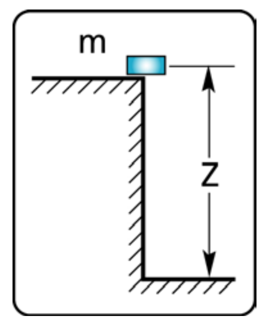
Kinetic Energy

The formula for kinetic energy is:



$$KE = \frac{1}{2}mv^2$$

The above equation states that the kinetic energy of a body is equal to one half the mass times the velocity squared.



Potential Energy

Another form of energy is potential energy:

$$PE = mgz$$

which means that the potential energy of a body is equal to the weight (mg) times the vertical distance (z) above a surface upon which the object would come to rest.

These two types of energies interact. For example, an object at the top of an incline has a certain amount of potential energy. After it's release, potential energy is gradually converted to kinetic energy. When it reaches the bottom, all the potential energy has converted to kinetic energy.

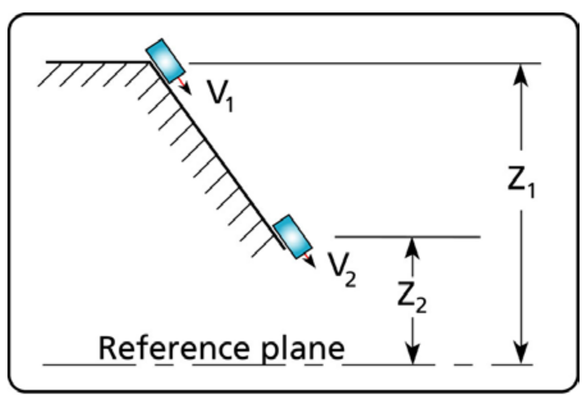


Figure 2-1 Transformation of potential energy to kinetic energy.

The principle of conservation of energy states that energy can neither be created or lost. Consequently, if one form of energy decreases then another form must increase. This allows us to make an energy balance that describes the energy variation of the object in Figure 2-1.

$$\Delta KE + \Delta PE = 0 \qquad [2\text{-}1]$$

In other words, equation [2-1] reads: the sum of the potential and kinetic energy variation between points 1 and 2 is equal to zero.

Energy levels are associated with positions within a system, position 1 is the top of the slope and position 2 the bottom.

$$\Delta PE = PE_1 - PE_2 \; and \; \Delta KE = KE_1 - KE_2$$

$$mg(z_1 - z_2) + \frac{1}{2}m(v_1{}^2 - v_2{}^2) = 0$$

$$v_2{}^2 - v_1{}^2 = 2g(z_1 - z_2) \qquad [2\text{-}2]$$

Equation [2-1] expresses the principle of conservation of energy for this system. This leads to equation [2-2] that describes how velocity changes with respect to height. The principle of conservation of energy makes it possible to account for all forms of energy in a system. The energies present in the system are in continuous change and the term delta (Δ) is used in equation [2-1] to indicate a change or variation in energy.

Thermodynamic properties

Thermodynamic properties are the different types of energies associated with a body (for example, potential, kinetic, internal or external). It is characteristic of a thermodynamic property is that its value is independent of the method or path taken to get from one value to another or from an initial state 1 to another state 2. The potential energy (PE) and the kinetic energy (KE) are **thermodynamic properties**. We require other energy quantities (work and heat) to account for real world conditions. These quantities are dependent on the path or method used to get from state 1 to state 2 (more about paths later).

2.1 CLOSED SYSTEMS AND INTERNAL ENERGY

The system shown in Figure 2-2 is a fluid contained in a sealed tube with its inlet connected to its outlet forming a closed system.

Internal Energy

All fluids have internal energy (U). If we apply a heat source to the system, the temperature, pressure and internal energy of the fluid will increase. Internal energy is the energy present at the molecular level of the substance.

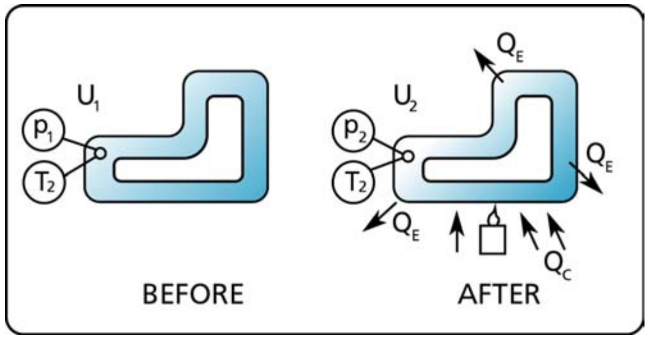


Figure 2-2 The relationship between internal energy and heat gain within a closed system.

Closed Systems

In a closed system no mass enters or leaves the boundary. What happens if we put a fluid within a closed environment such as a sealed container and apply heat? Pressure and temperature in the fluid will increase. The temperature rise indicates that the internal energy of the fluid has increased.

The heat source increases the internal energy of the fluid from U_1 to U_2 . The energy quantities present in this system are the internal energy (U) and the heat loss (Q). Therefore, the energy balance is:

$$Q = Q_C - Q_E = \Delta U = U_1 - U_2$$

Q_C is the quantity of heat absorbed by the fluid from the source and Q_E is the heat loss of the fluid to the environment. The internal energy has changed from its level at instant 1 to another level at instant 2 (after heat is applied). The internal energy (U) is a thermodynamic property.

2.2 CLOSED SYSTEMS, INTERNAL ENERGY AND WORK

Another way to increase the internal energy of a fluid is to do work on it by means of a pump. In this way, without applying heat, the work done on the fluid through the pump raises the internal energy of the fluid from U_1 to U_2. This causes the temperature of the fluid to increase, producing a heat loss Q_E to the environment.

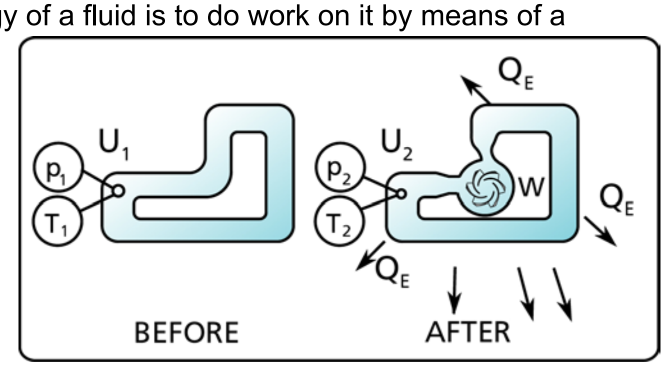


Figure 2-3 The relationship between internal energy, work and heat loss within a closed system.

In this situation, the energy balance is:

$$Q_E - W = \Delta U = U_1 - U_2$$

The sign convention is positive energy for heat leaving the system and negative energy for heat or work entering the system.

2.3 OPEN SYSTEMS AND ENTHALPY

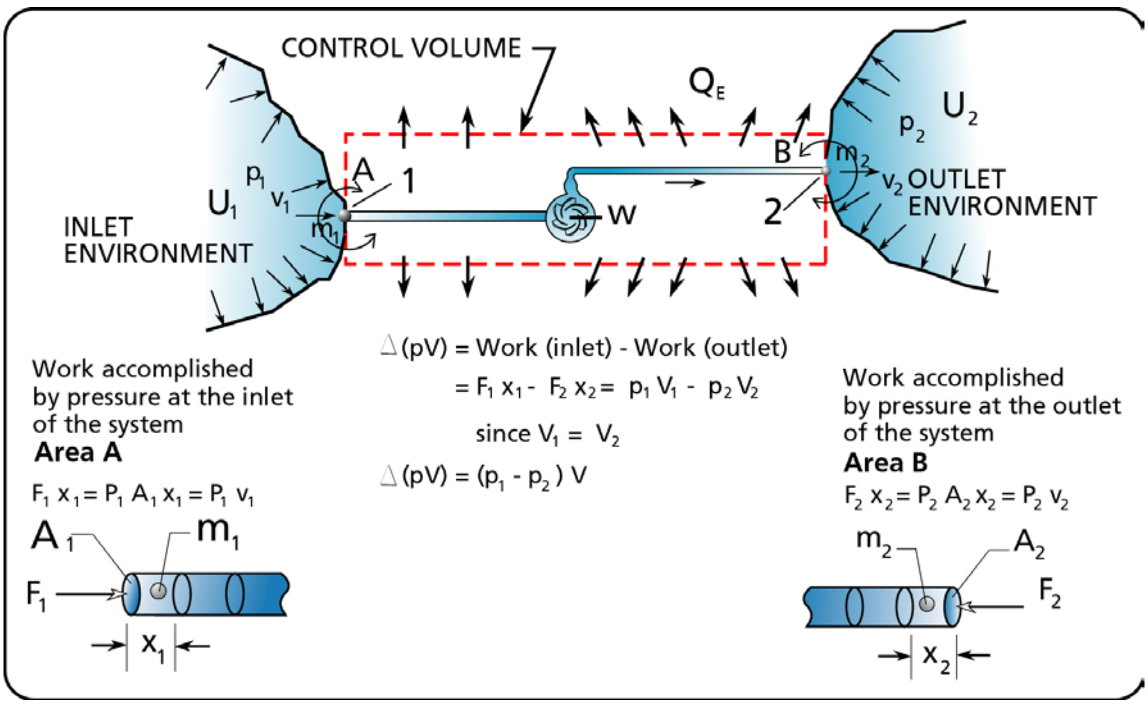


Figure 2-4 The relationship between internal energy, heat loss, work, and a pressurized environment for an open system.

An open system is one where mass is allowed to enter and exit. The mass entering the system displaces an equal amount of mass that exits. A clearly defined boundary, called the control volume, envelops the system and intersects the inlet and outlet. The control volume allows us to apply the principle of conservation of energy. Each unit of mass (m_1) and volume (V_1) that enters the system is subject to a pressure (p_1) at the inlet. The same is true of the mass leaving the system (m_2) of volume (V_2) and subjected to a pressure (p_2). The principle of conservation of mass requires that $m_1 = m_2$. In this book, we are dealing only with incompressible fluids where $V_1 = V_2$. The pressure at the inlet produces a certain quantity of work that helps push the fluid through the system. The pressure at the outlet produces work opposing that movement. The difference between these two work components is the work associated with pressure at the inlet and outlet of the system. This difference is the energy term Δ(pV) and is the only difference between the systems shown in Figure 2-3 vs. the system shown in Figure 2-4.

The energy balance is:

$$Q_E - W = \Delta U - \Delta(pV) = \Delta En$$

ΔU and Δ(pV) are terms that always occur together in an open system. For that reason their sum has been given the name enthalpy (En). Enthalpy is also a thermodynamic property.

2.4 OPEN SYSTEMS, ENTHALPY, KINETIC AND POTENTIAL ENERGY

In closed systems, the fluid particles move from one end of the container to another but they do not leave the container. It is impossible to have a net effect on the velocity and therefore on the kinetic energy of the system. In open systems, fluid particles move from the inlet to the outlet and leave the system. If the velocity varies between inlet and outlet, the kinetic energy varies. The pump is the energy source that compensates for the variation.

The same is true for potential energy in a closed system. A fluid particle that goes from the bottom of a container to the top will eventually return to the bottom. There can be no net effect on the potential energy of this system. In an open system, fluid particles can leave the system at a different elevation than they enter producing a variation in potential energy. The pump also compensates for this energy variation.

Potential Energy

The difference in vertical position at which the fluid enters and leaves the system causes the potential energy of a fluid particle to increase or decrease. The potential energy increases when the fluid leaves at a higher elevation than it enters. The pump provides energy to compensate for the increase in potential energy. If the fluid leaves the system at a lower elevation than it enters, the potential energy decreases and it is possible to convert this energy to useful work. This is what happens in a hydroelectric dam. A channel behind the dam wall brings water to a turbine located beneath the dam structure. The flow of water under pressure turns a turbine that is coupled to a generator producing electricity. The difference in elevation or potential energy of the fluid is the source of energy for the generator.

Kinetic Energy

There is often a velocity difference between the inlet and outlet of a system. Usually the velocity is higher at the outlet versus the inlet; this produces an increase in kinetic energy. The pump compensates for this difference in kinetic energy levels.

The complete energy balance for an open system is:

$$Q_E - W = \Delta En + \Delta KE + \Delta PE \qquad [2\text{-}3]$$

Equation [2-3] has the thermodynamic properties on the right-hand side. The heat loss (Q_E) and the work done on the system (W), which are not thermodynamic properties, are on the left-hand side of the equation. To determine Q_E and W, it is necessary to know how the heat is produced (i.e. path that the fluid must take between inlet and outlet) and how the work is done.

2.5 WORK DONE BY THE PUMP

The role of a pump is to provide sufficient pressure to move fluid trough the system at the required flow rate. This energy compensates for the energy losses due to friction, elevation, velocity and pressure differences between the inlet and outlet of the system. This book does not deal with the method by which a pump can pressurize a fluid within its casing. The reader is referred to reference 15, a very detailed treatment of this subject. For our purpose, the pump is a black box whose function is to increase the fluid pressure at a given flow rate.

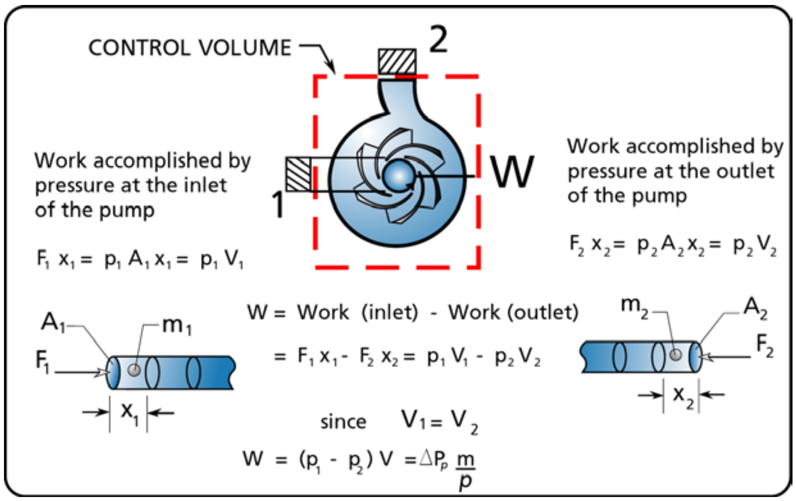


Figure 2-5 The work provided by the pump.

2.6 FLUID AND EQUIPMENT FRICTION LOSS

The total heat loss (Q_E) is the sum of fluid friction loss and equipment friction loss.

$$Q_E = Q_F + Q_{EQ}$$

Fluid Friction Loss (Q_F)
How is it possible to go from a nebulous concept such as fluid friction to the pressure drop due to friction? Friction occurs within the fluid and between the fluid and the walls.. A detailed method of fluid friction calculation is presented in Chapter 3. Friction in fluids produces heat (Q_F) which dissipates to the environment. The temperature increase is negligible. If we can estimate the friction force, we know that we must supply an equal and opposite force to overcome it.

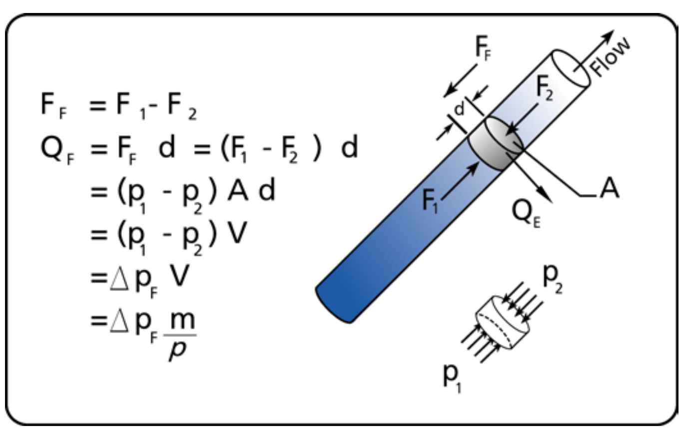


Figure 2-6 Heat loss due to fluid friction within pipes.

The energy corresponding to the heat loss (Q_F) must be supplied by the pump. The net force (F_F) required to balance the friction force is F_1-F_2 These forces are the result of the action of pressures p_1 and p_2 (see Figure 2-6). The difference between p_1 and p_2 is the pressure differential or pressure drop due to friction for a given length of pipe. Many publications (reference 1 and 8 for example) provide pressure head drop values in the form of tables. The pressure head drop can also be calculated by the Colebrook equation, which we will discuss in chapter 3.

Equipment Friction Loss (Q_{EQ})

Many different types of equipment are present in typical industrial systems. Control valves, filters, etc., are examples of equipment that have an effect on the fluid. Because of the great variety of different equipment that can be installed, it is not possible to analyze everyone of them. However we can quantify the effect of the equipment by determining the pressure drop across the equipment much as we did with the pressure drop across the pump. Again it is not necessary to know exactly what happens inside the equipment to be able estimate this effect. Pressure drop (difference between inlet and outlet pressure) is the outward manifestation of the effect of equipment. This pressure drop produces heat (Q_{EQ}) which is then lost through the fluid to the environment. The heat loss is calculated in the same manner as the pump work (W).

$$Q_{EQ} = \Delta p_{EQ} \frac{m}{\rho}$$

The product supplier normally makes available the amount of pressure drop for the equipment at various flow rates in the form of charts or tables.

2.7 THE CONTROL VOLUME

Up to now we have been using control volumes without defining their purpose. A control volume is an imaginary boundary that surrounds a system and intersects all inlets and outlets. This makes it possible to apply the principle of conservation of mass. The boundary of the system is determined by appropriately locating the inlet and the outlet. The term “appropriate” refers to locating the boundary at points where the conditions (pressure, elevation and velocity) are known. The control volume encompasses all internal and external the energy sources affecting the system. This makes it possible to apply the principle of conservation of energy.

How is the control volume positioned in real systems? Figure 2-7 shows a typical industrial process. Suppose that we locate the inlet (point 1) on the inlet pipe as shown in Figure 2-7. Is this reasonable? Where exactly should we locate it, will any position due? Clearly no. Point I could be located anywhere on the feed line to the suction tank. The problem with locating point 1 on the inlet line is precisely that the location is indeterminate.

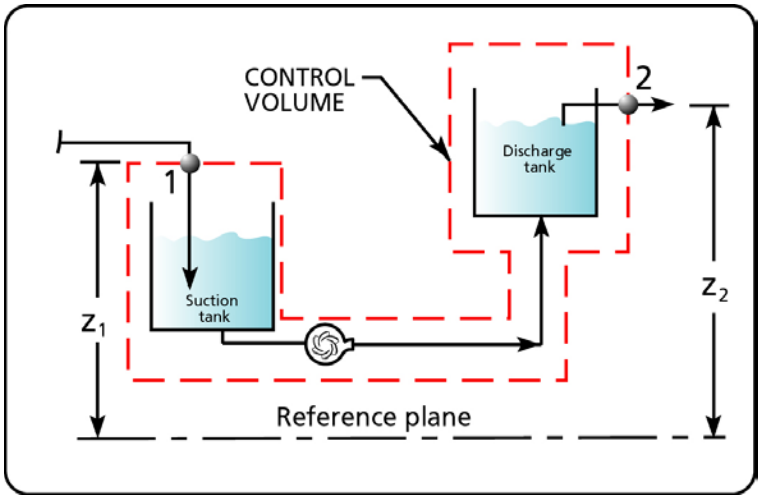


Figure 2-7 Incorrect positioning of the control volume in a pumping system.

However, consider Figure 2-8. The pressure head at the inlet of the pump is proportional to the elevation of the suction tank fluid surface. We locate the inlet of the system (point 1) on the suction tank fluid surface since any change in the elevation of point 1 will affect the pump. The inlet pipe is the means by which we supply the pump with enough fluid to operate. As far as this system is concerned, the inlet feed pipe is irrelevant. This is because the pressure head at the inlet of the pump (or any point within the system) is dependent on the level of the suction fluid tank surface. The other reason that requires us to locate point 1 on the liquid surface of the suction tank is that the control volume must contain all the energy sources that affect the system and if the suction and discharge tanks are open to atmosphere than this pressure will be present at points 1 and 2 ($H_1 = H_2 = 0$ = atmospheric pressure).

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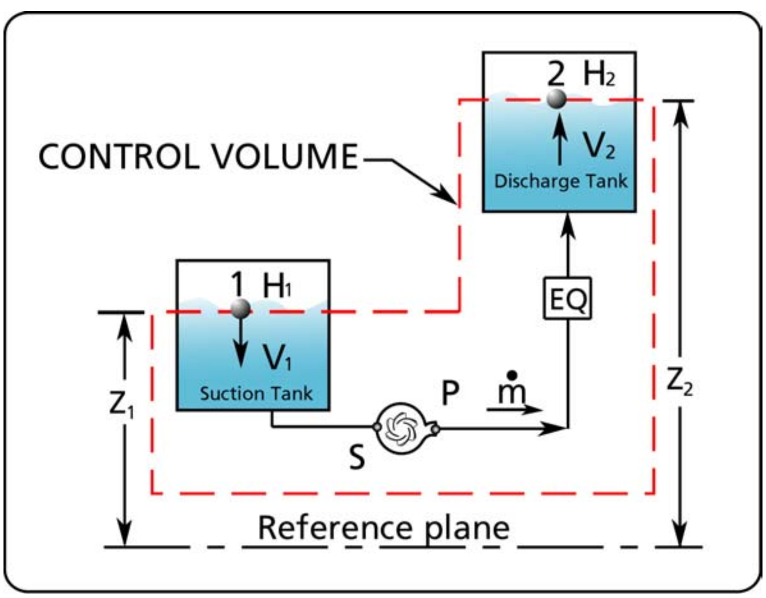

Figure 2-8 Proper positioning of the intersections of the control volume with a pumping system.

This leads to a representation of a generalized system that we will use from now on. A general system must take into account the possibility that the suction and discharge tanks are pressurized. The terms H_1 and H_2 in Figure 2-8 represent respectively the suction and discharge pressure head at the fluid surfaces of these tanks.

A general system must take into account the possibility that the system contains no tanks and that points 1 and 2 are connections to pipe lines of other systems. The only difference between a tank and a pipe is that a tank can contain a much greater volume of fluid. Therefore, whether the system has tanks or not is immaterial. However the conditions (i.e. pressure, velocity and elevation) of the inlet and outlet of the system must be known. It will always be possible to disconnect our system from another if the pressure head, velocity and elevation (H , v and z) at the connection point is known (more about disconnecting interconnected systems later in this chapter). Therefore, Figure 2-8 can represent all possible situations.

2.8 THE DETERMINATION OF TOTAL HEAD FROM THE ENERGY BALANCE

We now have all the background information necessary to determine the work required of the pump and therefore the Total Head. First, we calculate the energy lost in friction due to fluid movement through pipes (Q_F) and equipment (Q_{EQ}.). Next, calculate the energy variations due to conditions at the inlet and outlet.

1. Potential energy variation (ΔPE) is associated with the difference in height of a fluid element between the outlet and inlet of the system.

2. Kinetic energy variation (ΔKE) is associated with the variation in velocity of a fluid element between the outlet and inlet of the system.

3. Enthalpy variation (ΔEn) consists of two components $\Delta En = \Delta U + \Delta(pV)$. The first is the internal energy (ΔU) of the fluid. In most common fluid transfer situations, there is no significant temperature change and therefore no significant internal energy change ($\Delta U = U_1 - U_2 = 0$). The other component of enthalpy Δ(pV) is the energy associated with the difference in pressure at the inlet and outlet of the system.

The difference between the inlet and outlet energies added to the total friction loss (Q_E) gives the resultant work W that must be supplied by the pump.

2.9 SYSTEM OR TOTAL HEAD EQUATION FOR A SINGLE INLET - SINGLE OUTLET SYSTEM

The simplest case we can consider is a system with one inlet and one outlet. The first law of thermodynamics or the principle of conservation of energy for open systems states:

$$\dot{Q}_E - \dot{W} = \Delta \dot{E}n + \Delta \dot{K}E + \Delta \dot{P}E \qquad [2\text{-}4]$$

Equation [2-4] expresses the rate of variation of the energy terms instead of the energy variation as in equation [2-3]. This allows us to balance mass flow rates instead of mass between the inlet and outlet of the system. In Figure 2-8, the box with the letters EQ represents the effect of all the equipment present in the line between points 1 and 2. H_1 and H_2 are the pressure heads present at the suction tank and the discharge tank fluid surface respectively. z_1 and v_1 are respectively the elevation and the velocity of a fluid particle at the inlet of the system, and z_2 and v_2 are the same variables for a fluid particle at the outlet.

$$ENERGY\ RATE = v\,\Delta F = \dot{V}\Delta p = \frac{\dot{m}}{\rho}\Delta p$$

The following equations are all based on the above which is the work or energy rate required to move a body of mass m at a velocity v. $\dot{V}$ and $\dot{m}$ are respectively the volumetric and mass flow rate. Δp is the pressure differential and ρ is the fluid density.

The energy rate of heat loss $(\dot{Q}_E)$

The rate of heat transfer $(\dot{Q}_E)$ is the heat generated in the system by fluid friction in the pipes and through the equipment:

$$\dot{Q}_E = \frac{\dot{m}}{\rho}\left(\Delta p_{F1-2} + \Delta p_{EQ1-2}\right) \quad [2\text{-}5]$$

where Δp_{F1-2} is the pressure drop associated with fluid friction for a fluid particle traveling between points 1 and 2. Δp_{EQ1-2} is the sum of all pressure drops produced by all of the equipment between the same points.

The rate of mechanical work $(\dot{W})$

Similarly, the rate of mechanical energy introduced into the system, such as supplied by a pump is:

$$\dot{W} = \frac{\dot{m}}{\rho}\Delta p_P \quad [2\text{-}6]$$

where Δp_P is the difference in pressure between the discharge and the suction of the pump (points P and S in Figure 2-8).

The rate of enthalpy variation $(\Delta\dot{E})$

The rate of enthalpy variation is composed of the rate of internal energy variation ($\Delta\dot{U}$), and the difference in pressure energy between the inlet or outlet of the system $\Delta(p\dot{V})$. $\Delta\dot{U}$ is normally zero or negligible in most fluid transfer situations, therefore:

$$\Delta\dot{E} = \Delta\dot{U} + \Delta(p\dot{V}) = \dot{U}_1 - \dot{U}_2 + (p_1\dot{V}_1 - p_2\dot{V}_2) = p_1\dot{V}_1 - p_2\dot{V}_2 \text{ since } \dot{U}_1 - \dot{U}_2 = 0$$

also for incompressible fluids $\dot{V}_1 = \dot{V}_2$, therefore

$$\Delta \dot{E}n = \dot{V}(p_1 - p_2) = \frac{\dot{m}}{\rho}(p_1 - p_2) \qquad [2\text{-}7]$$

where p_1 is the pressure at the suction tank liquid surface and p_2 is the pressure at the discharge tank liquid surface.

The rate of kinetic energy variation $(\Delta \dot{K}E)$

Kinetic energy is the energy associated with the velocity (v) of a body of mass (m).

The rate of kinetic energy variation of the system is:

$$\Delta \dot{K}E = \frac{1}{2g_c}\dot{m}(v_1{}^2 - v_2{}^2) \qquad [2\text{-}8]$$

where v_1 is the velocity of a particle at the surface of the suction tank or the system inlet velocity. v_2 is the velocity of a particle at the surface of the discharge tank or the system outlet velocity. The constant (g_c) is required to make the units consistent in the FPS system.

The rate of potential energy variation $(\Delta \dot{P}E)$

Potential energy is the energy associated with the vertical position z_2 or z_1 of a mass (m) subject to the influence of a gravity field. The change of potential energy of fluid particles in the system is:

$$\Delta \dot{P}E = \dot{m}\frac{g}{g_c}(z_1 - z_2) \qquad [2\text{-}9]$$

where z_1 and z_2 are respectively the elevation of a particle on the fluid surface of the suction and discharge tank. Elevations such as z_1 and z_2 are often taken with respect to a DATUM or reference plane.

By substituting equations [2-5] to [2-9] in equation [2-4] and dividing by $\dot{m}\frac{g}{g_c}$, we obtain:

$$\frac{(\Delta p_{F1-2} + \Delta p_{EQ1-2})}{\rho\frac{g}{g_c}} - \frac{\Delta p_P}{\rho\frac{g}{g_c}} = \frac{(p_1 - p_2)}{\rho\frac{g}{g_c}} + \frac{1}{2g}(v_1{}^2 - v_2{}^2) + (z_1 - z_2) \qquad [2\text{-}10]$$

Equation [2-10] is Bernoulli's equation with the pump pressure increase (Δp_P) and fluid (Δp_{F1-2}) and equipment (Δp_{EQ1-2}) friction terms added. Pressure can be expressed in terms of fluid column height or pressure head as demonstrated in chapter 1.

$$p = \frac{\rho g H}{g_c} \qquad [2\text{-}11]$$

All pressure terms are replaced by their corresponding fluid column heights with the use of equation [2-11], for example $p_1 = \frac{\rho g H_1}{g_c}$, $\Delta p_{F1-2} = \frac{\rho g \Delta H_{F1-2}}{g_c}$, etc.). The constant g_c cancels out.

The Total Head is:

$$\boxed{\Delta H_P(ft\ fluid) = (\Delta H_{F1-2} + \Delta H_{EQ1-2}) + \frac{1}{2g}({v_2}^2 - {v_1}^2) + z_2 + H_2 - (z_1 + H_1)} \qquad [2\text{-}12]$$

The unit of Total Head (ΔH_P) is feet of fluid. The pump manufacturers always express the Total Head (ΔH_P) in feet of water, is a correction required if the fluid is other than water? Do we need to convert feet of fluid to feet of water? The terms in equations [2-10] and [2-12] are in energy per pound of fluid, or lbf-lbf/lbf; which is the same as feet. Since head is really energy per unit weight, the density of the fluid becomes irrelevant (or in other words 1 pound of water is the same as 1 pound of mercury). However, we will see later that the motor power required to move the fluid at a certain rate does required that the density of the fluid be considered (see Chapter 4).

The pump manufacturers test the performance of their pumps with water. The capacity of a centrifugal pump is negatively affected by the fluid's viscosity; therefore the three major performance parameters of a pump (total head, flow rate and efficiency) will have to be corrected for fluids with a viscosity higher than water (see reference 1 and the web site www.fluidedesign.com).

EXAMPLE 2.1 – CALCULATE THE TOTAL HEAD FOR A TYPICAL PUMPING SYSTEM

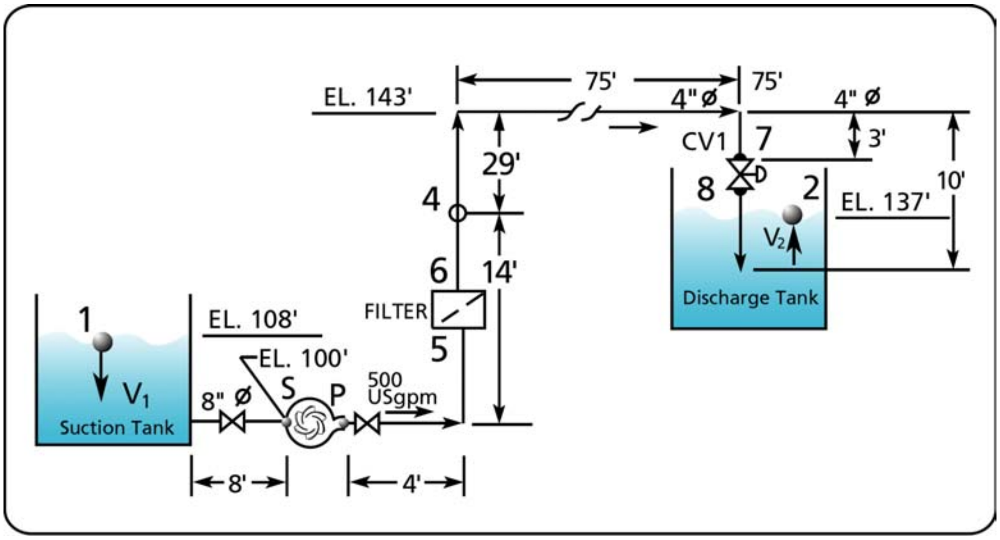

Figure 2-9 An example of a calculation for Total Head of a typical pumping system.

Our first example will take us through the ins and outs of a Total Head calculation for a simple pumping system. The fluid is water at 60 °F.

The equation for Total Head is:

$$\Delta H_P = \Delta H_{F1-S} + \Delta H_{FP-4} + \Delta H_{F4-2} + \Delta H_{EQ1-S} + \Delta H_{EQP-4} + \Delta H_{EQ4-2} + \frac{1}{2g}(v_2^{\ 2} - v_1^{\ 2}) + z_2 + H_2 - (z_1 + H_1)$$

Point 4 is not essential to solve this problem but required for comparison purposes in example 2.2 where a branch is added at point 4.

Friction Head Difference Pipe & Fittings

Since the pipe diameter changes between points 1 and 2, it is necessary to determine the friction occurring between points 1 and S and points P and 2.

POINT 1 TO S

Pipe friction loss between points 1 and S.

$$\Delta H_{F1-2} = \Delta H_{F1-S} + \Delta H_{FP-2}$$

The friction loss term ΔH_{F1-S} is made up of fluid friction and fittings friction.

Pipe friction = Fluid friction (pipe) + Fluid friction (fittings)

$$\Delta H_{F1-S} = \Delta H_{FP1-S} + \Delta H_{FF1-S} \; and \; \Delta H_{FP-2} = \Delta H_{FPP-2} = \Delta H_{FFP-2}$$

where ΔH_{FP1-S} is the fluid friction between points 1 and S and ΔH_{FF1-S} is the friction produced by fittings (for example elbows, isolation valves, etc.) between the same two points. From the tables (reference 1 or 8). for a 8" dia. Pipe and a flow rate of 500 USGPM:

$\Delta H_{FP1-S}/L$ = 0.42 ft/100 ft of pipe.

$$\Delta H_{FF1-S} = \frac{\Delta H_{FP}}{L} \times \frac{L_{1-S}}{100} = 0.42 \times \frac{8}{100} = 0.03 \, ft$$

Fittings Friction Loss between points 1 and S

In this portion of the line there is (1) 8" butterfly valve, from the tables (reference 1 or 8) K = 0.25

$$\Delta H_{FF1-S} = \frac{K {v_S}^2}{2g} = \frac{0.25 \times 3.19^2}{2 \times 32.17} = 0.04 \, ft$$

The pipe and fittings friction loss between points 1 and S is:

$\Delta H_{F1-S} = \Delta H_{FP1-S} + \Delta H_{FF1-S}$ = 0.03 + 0.04 = 0.07 ft

POINT P TO 4.

Pipe friction loss between points P and 4.

From the tables (reference 1 or 8), for a 4" dia. Pipe at a flow rate of 500 USGPM:
ΔH_{FPP-4} /L= 13.1 ft/100 ft of pipe.

$$\Delta H_{FPP-4} = \frac{\Delta H_{FP}}{L} \times \frac{L_{P-4}}{100} = 13.1 \times \frac{18}{100} = 2.3 \, ft$$

Fittings Friction Loss between points P and 4

In this portion of the line there is:
4” dia. elbow, from the tables (reference 1 or 8) K = 0.25;
(1) 4”Φ isolation (butterfly) valve; from the tables (reference 1 or 8) K_{VAL} = 0.25

$$\Delta H_{FFP-4} = \frac{(K_{ELB} + K_{VAL}) \times v_4^{\ 2}}{2g} = \frac{(0.25 + 0.25) \times 12.76^2}{2 \times 32.17} = 1.3\ ft$$

The pipe and fittings friction between points P and 4 is:

$\Delta H_{FP-4} = \Delta H_{FPP-4} + \Delta H_{FFP-4}$ = 2.3 + 1.3 = 3.6 ft

POINT 4 TO 2

Pipe Friction Loss between points 4 and 2.

From the tables (reference 1 or 8), for a 4"dia. pipe at a flow rate of @ 500 USGPM:

ΔH_{FPP-2} /L= 13.1 ft/100 ft of pipe.

$$\Delta H_{FP4-2} = \frac{\Delta H_{FP}}{L} \times \frac{L_{4-2}}{100} = 13.1 \times \frac{114}{100} = 14.9\ ft$$

Fittings Friction Loss between points 4 and 2

In this portion of the line there is (2) 4"dia. elbows, from the tables (reference 1 or 8) K_{ELB} = 0.25.

$$\Delta H_{FF4-2} = \frac{2 \times K_{ELB} \times v_2^{\ 2}}{2g} = \frac{2 \times 0.25 \times 12.76^2}{2 \times 32.17} = 1.3\ ft$$

The pipe and fittings friction between points 4 and 2 is:

$\Delta H_{F4-2} = \Delta H_{FP4-2} + \Delta H_{FF4-2}$ = 14.9 + 1.3 = 16.2 ft

Equipment Pressure Head Difference

POINT 1 TO S

There is no equipment in this portion of the line.

$\Delta H_{EQ1-S} = 0$

POINT P TO 4

The filter between point P and 4 is considered a piece of equipment. The filter manufacturer's technical data states that this filter will produce a 5 psi pressure drop at a flow rate of 500 USGPM. For water 5 psi is equivalent to 11.5 ft of head (5 X 2.31, see equation [1-5]).

$\Delta H_{EQP-4} = \Delta H_{EQFIL} = 11.5$ ft

POINT 4 TO 2

The control valve between point P and 4 is considered a piece of equipment. The control valve will have a head drop of 10 ft of fluid. The justification for this will be given later in Chapter 3.

$\Delta H_{EQ4-2} = \Delta H_{EQCV} = 10$ ft

Velocity Head Difference

The velocity head difference between points l and 2 is zero, since v_1 and v_2 are very small.

$$\Delta H_v = \frac{1}{2g}(v_2^{\ 2} - v_1^{\ 2}) = 0$$

Static Head

Both tanks are not pressurized therefore $H_1 = H_2 = 0$.

The elevation difference $z_2 - z_1 = 137 - 108 = 29$ ft

Total Head

Once again the equation for Total Head is:

$$\Delta H_P = \Delta H_{F1-S} + \Delta H_{FP-4} + \Delta H_{F4-2} + \Delta H_{EQ-S} + \Delta H_{EQP-4} + \Delta H_{EQ4-2} + \frac{1}{2g}(v_2^{\ 2} - v_1^{\ 2}) + z_2 + H_2 - (z_1 + H_1)$$

By substituting the values previously calculated into the above equation:

ΔH_P= 0.07 + 3.6 + 16.2 + 0 + 11.5 + 10+ 0 + 137 + 0 - (108 + 0) = 70.4 ft

The result of the calculation indicates that the pump will require a Total Head of 70.4 ft to deliver 500 USGPM. **How can we guarantee that the pump will deliver 500 USGPM at 70 ft of Total Head?** The answer is in Chapter 4, section 4.5.

2.10 METHOD FOR DETERMINING THE PRESSURE HEAD AT ANY LOCATION

The system configuration in Figure 2-10 illustrates how drastically the pressure can vary in a simple pumping system. Why do we need to calculate the pressure at a specific location in the system? One reason is the pressure head just before the control valve is a parameter required to size the valve. It is calculated by the method described below. In addition, the system may have other equipment where the inlet pressure must be known to ensure proper operation.

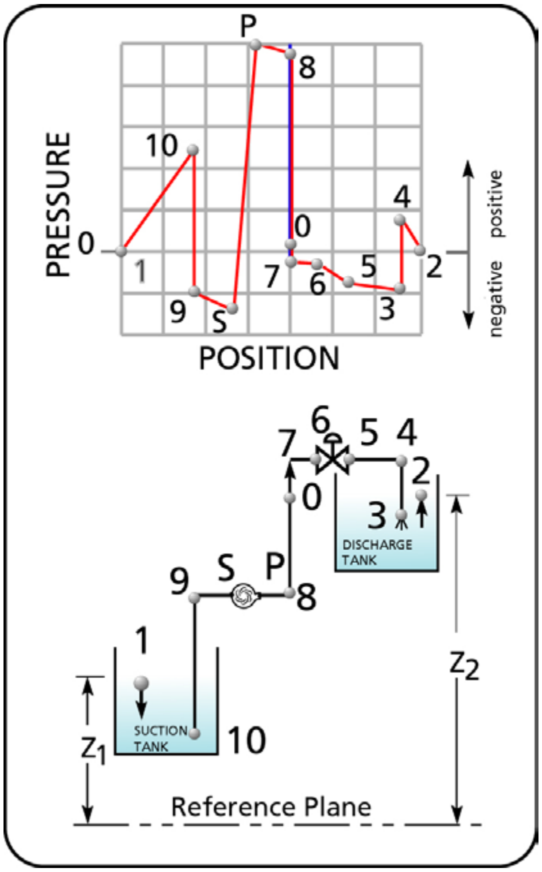

Figure 2-10 The pressure head variation within a typical pumping system.

A. The pressure head at any location on the discharge side of the pump.

First, calculate the Total Head (ΔH_P) for the complete system using equation [2-12]. Then draw a control volume (see Figure 2-11) to intersect point X and point 1. Point X can be located anywhere between P and 2. The system equation [2-12] is then applied with point X as the outlet instead of point 2 and solved for the unknown variable H_x instead of ΔH_P.

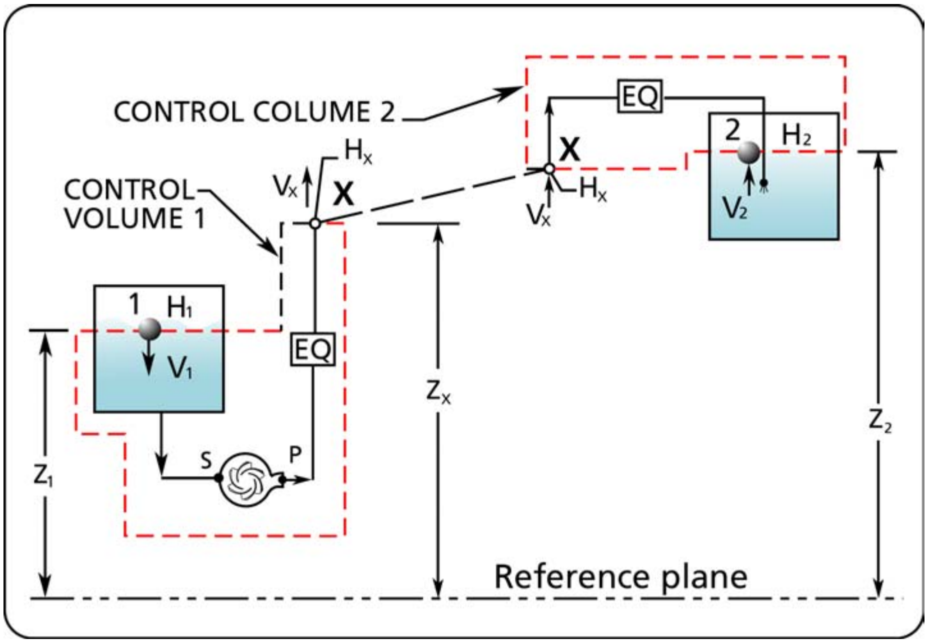

Figure 2-11 Using control volumes to determine the pressure head anywhere after the pump discharge

From equation [2-12], the Total Head for the complete system is:

$$\Delta H_P = \Delta H_{F1-2} + \Delta H_{EQ1-2} + \frac{1}{2g}(v_2^{\,2} - v_1^{\,2}) + z_2 + H_2 - (z_1 + H_1)$$

Equation [2-12] is applied with the following changes: all terms with subscript 2 are replaced with the subscript X, $H_2=H_X$, $\Delta H_{F1-2} = \Delta H_{F1-X}$, $\Delta H_{EQ1-2} = \Delta H_{EQ1-X,}$ $v_2 = v_{X,}$ $z_2 = z_{X.}$

The unknown term Hx is isolated on one side of the equation:

$$H_X = \Delta H_P - (\Delta H_{F1-X} + \Delta H_{EQ1-X}) + \frac{1}{2g}(v_1^{\,2} - v_X^{\,2}) + z_1 + H_1 - z_X \qquad [2\text{-}13]$$

The unknown pressure head H_x can also be determined by using that part of the system defined by control volume 2 (see Figure 2-11) by using the same reasoning as above, in equation [2-12], $\Delta H_P = 0$ and all terms with subscripts 1 replaced with X.

Therefore, $H_1=H_X$, $\Delta H_{F1-2} = \Delta H_{FX-2}$, $\Delta H_{EQ1-2} = \Delta H_{EQX-2,}$ $v_1= v_{X,}$ $z_1 = z_X$

$$H_X = (\Delta H_{FX-2} + \Delta H_{EQX-2}) + \frac{1}{2g}(v_2^{\,2} - v_X^{\,2}) + z_2 + H_2 - z_X \qquad [2\text{-}14]$$

We now have two methods for determining the pressure head at any location. We can use one equation to verify the results of the other. The calculation for H_X is often quicker using equation [2-14].

Note that in equation [2-14] the value of ΔH_P is not required to calculate H_x.

B. The pressure head at any location on the suction side of the pump

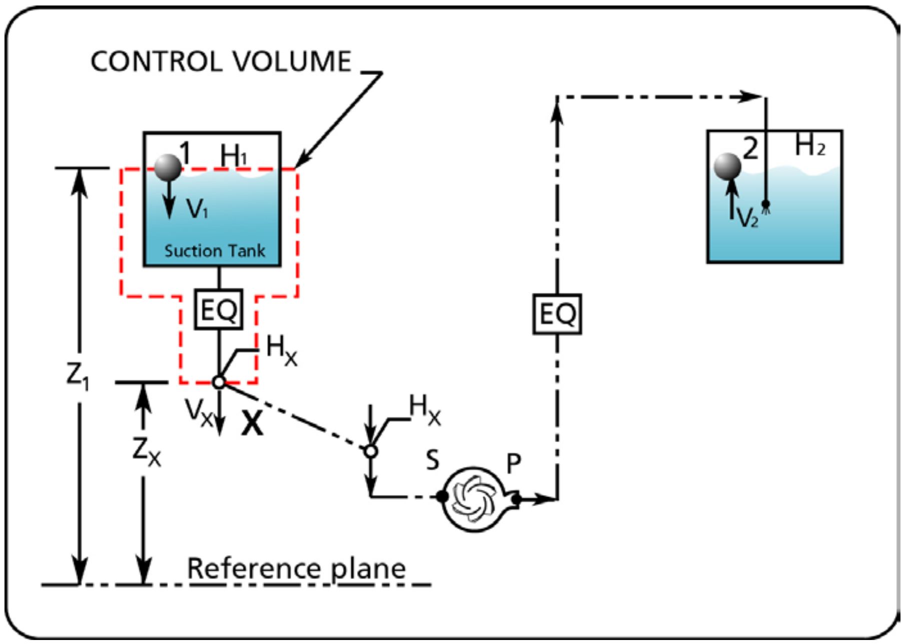

Figure 2-12 Using control volumes to determine the pressure head anywhere before the pump inlet.

We can determine the pressure head anywhere on the suction side of the pump by the same method. In this case, $\Delta H_P = 0$ since there is no pump within the control volume (see Figure 2-12). $\Delta H_P = 0$ and subscript 2 is replaced with subscript X in equation [2-12]. We obtain:

$$H_X = -(\Delta H_{F1-X} + \Delta H_{EQ1-X}) + \frac{1}{2g}(v_1^{\,2} - v_X^{\,2}) + z_1 + H_1 - z_X \qquad [2\text{-}15]$$

The velocity v_X must be the same as would occur at point X in the complete system. In chapter 3, we will use equation [2-15] to calculate the available Net Positive Suction Head of the pump.

EXAMPLE 2.2 – CALCULATE THE PRESSURE HEAD AT THE INLET OF A CONTROL VALVE

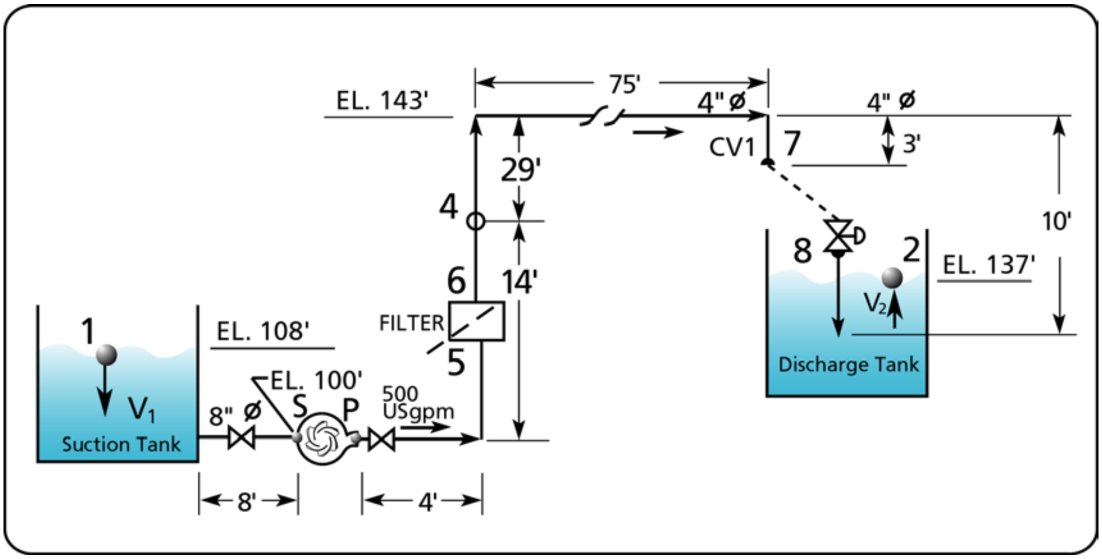

Figure 2-13 Example of the calculation of the pressure head at the inlet of a control valve.

It is often required to know the pressure head just ahead of a piece of equipment such as a control valve. We will apply the methods in the previous section to calculate the pressure head just ahead of the control valve in example 2.1.

CALCULATE THE PRESSURE HEAD AT POINT 7 USING EQUATION [2-13]

Applying equation [2-13] where point X is now point 7 (X = 7) and ΔH_P = 70.4 from the calculation in example 2.1.

$$H_7 = 70.4 - (\Delta H_{F1-S} + \Delta H_{EQ1-S}) - (\Delta H_{FP-7} + \Delta H_{EQP-7}) + \frac{1}{2g}(v_1^2 - v_7^2) + z_1 + H_1 - z_7$$

Friction Head Difference - Pipe and Fittings

POINT 1 TO S

From example 2.1: ΔH_{F1-S} = 0.07 ft

POINT P TO 7

$\Delta H_{FP\text{-}7} = \Delta H_{FPP\text{-}7} + \Delta H_{FFP\text{-}7}$

Pipe friction loss between point P and 7.

From the tables (reference 1 or 8), for a 4"dia. at a flow rate of 500 USGPM:

$\Delta H_{FPP\text{-}7}$ /L= 13.1 ft/100 ft of pipe.

$$\Delta H_{FPP-7} = \frac{\Delta H_{FP}}{L} \times \frac{L_{P-7}}{100} = 13.1 \times \frac{125}{100} = 16.4\ ft$$

Fittings Friction Loss between points P and 7

In this portion of the line there is:
(3) 4" dia. elbow, K_{ELB} = 0.25;
(1) 4" dia. isolation (butterfly) valve, K_{VAL} = 0.25.

$$\Delta H_{FFP-7} = \frac{(3 \times K_{ELB} + K_{VAL}) \times v_7^{\,2}}{2g} = \frac{(3 \times 0.25 + 0.25) \times 12.6^2}{2 \times 32.17} = 2.5\ ft$$

The pipe and fittings friction between points P and 7 is:

$\Delta H_{FP\text{-}7} = \Delta H_{FPP\text{-}7} + \Delta H_{FFP\text{-}7} = 16.4 + 2.5 = 18.9$ ft.

Equipment Pressure Head Difference

POINT 1 TO S

$\Delta H_{EQ1\text{-}S} = 0$

POINT P TO 7

The filter between point P and 4 has a pressure head drop of:
$\Delta H_{EQP\text{-}7} = \Delta H_{EQFIL} = 11.5$

Velocity Head Difference

The velocity head difference between points 1 and 7 is:

$$\Delta H_v = \frac{1}{2g}(v_1^{\,2} - v_7^{\,2}) = \frac{1}{2g}(0 - 12.76^2) = -2.5\, ft$$

Static Head

The suction tank is not pressurized, therefore $H_1 = 0$.

The elevation difference $z_1 - z_7 = 108 - 140 = -32$ ft

The pressure head at point 7

Again the equation for the pressure head at point 7 is:

$$H_7 = 70.4 - (\Delta H_{F1-S} + \Delta H_{EQ1-S}) - (\Delta H_{FP-7} + \Delta H_{EQP-7}) + \frac{1}{2g}(v_1^{\,2} - v_7^{\,2}) + z_1 + H_1 - z_7$$

By substituting the values calculated into the above equation we obtain:

$H_7 = 70.4 - 0.07-(18.9 + 11.5) - 2.5 + 108 + 0 -140 = 5.4$ ft

CALCULATE THE PRESSURE HEAD AT POINT 7 USING EQUATION [2-14]

This is a good opportunity to verify our calculation for H_7 using equation [2-14]. Substituting 7 for X, the equation is:

$$H_7 = (\Delta H_{F7-2} + \Delta H_{EQ7-2}) + \frac{1}{2g}(v_2^{\,2} - v_7^{\,2}) + z_2 + H_2 - z_7$$

Friction Head - Pipe and Fittings

POINT 7 TO 2

$\Delta H_{F7-2} = \Delta H_{FP7-2} + \Delta H_{FF7-2}$

Pipe friction loss between point 7 and 2.

From the tables (reference 1 or 8), for a 4" dia. at a flow rate of 500 USGPM:

ΔH_{FP7-2} /L= 13.1 ft/100 ft of pipe.

$$\Delta H_{FP7-2} = \frac{\Delta H_{FP}}{L} \times \frac{L_{7-2}}{100} = 13.1 \times \frac{7}{100} = 0.92\ ft$$

Fittings Friction Loss between points 7 and 2

In this portion of the line, there is no fittings. Therefore $\Delta H_{FF7-2} = 0$

The pipe and fittings friction between points 7 and 2 is:

$\Delta H_{F7-2} = \Delta H_{FP7-2} + \Delta H_{FF7-2} = 0.92 + 0 = 0.92$ ft.

Equipment Pressure ahead Difference

POINT 7 TO 2

As previously stated, we allow a pressure head drop of 10 ft for the control valve.

$\Delta H_{EQ7-2} = 10$ ft

Velocity Head Difference

The velocity head difference between points 7 and 2 is:

$$\Delta H_v = \frac{1}{2g}(v_2^2 - v_7^2) = \frac{1}{2g}(0 - 12.76^2) = -2.5\ ft$$

Static Head

The discharge tank is not pressurized, therefore $H_2 = 0$.

The elevation difference $z_2 - z_7 = 137 - 140 = -3$ ft

The pressure head at point 7

Again the equation for the pressure head at point 7 is:

$$H_7 = (\Delta H_{F7-2} + \Delta H_{EQ7-2}) + \frac{1}{2g}(v_2^{\,2} - v_7^{\,2}) + z_2 + H_2 - z_7$$

By substituting the values calculated into the above equation:

H_7 = 0.9 + 10 - 2.5 + 137 + 0 - 140 = 5.3 ft - the same result as previously.

2.11 SYSTEM OR TOTAL HEAD EQUATION FOR A SINGLE INLET - DOUBLE OUTLET SYSTEM

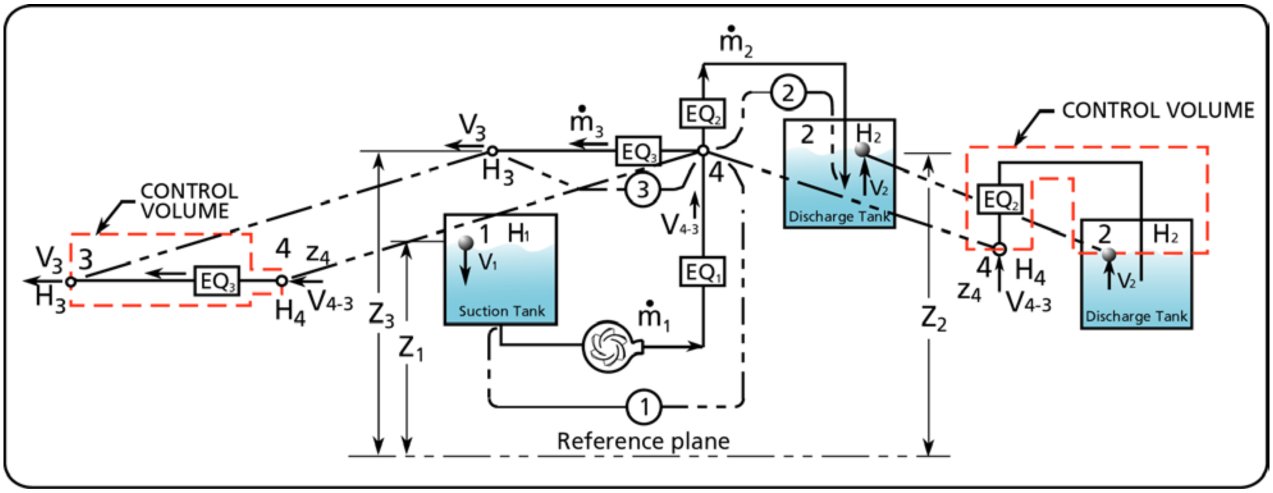

Figure 2-14 The use of control volumes to determine the Total Head for a single inlet – double outlet system.

Industrial systems often have more than one outlet (see Figure 2-14).

Equation [2-4] is used as the basis for the analysis of the system using the principle of conservation of mass flow rate for a steady state system.

$$\dot{Q}_E - \dot{W} = \Delta\dot{E}n + \Delta\dot{K}E + \Delta\dot{P}E$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

The same logic used to deduce equation [2-12] can be used for a system with multiple inlets and outlets. $\dot{Q}_E$ is the sum of all the friction heat losses and equipment losses. $\dot{W}$ is the work supplied by the pump. $\Delta\dot{E}n$ is the sum of the inlet minus the sum of the outlet enthalpies. $\Delta\dot{K}E$ is the sum of the inlet minus the sum of the outlet kinetic energies. $\Delta\dot{P}E$ is the sum of the inlet minus the sum of the outlet potential energies. The energy balance is:

$$\frac{\dot{m}_1}{\rho}\left(\Delta p_{F1-4} + \Delta p_{EQ1-4}\right) + \frac{\dot{m}_2}{\rho}\left(\Delta p_{F4-2} + \Delta p_{EQ4-2}\right) + \frac{\dot{m}_3}{\rho}\left(\Delta p_{F4-3} + \Delta p_{EQ4-3}\right) - \frac{\dot{m}_1}{\rho}\Delta p =$$

$$\frac{\dot{m}_1}{\rho}p_1 - \left(\frac{\dot{m}_2}{\rho}p_2 + \frac{\dot{m}_3}{\rho}p_3\right) + \frac{\dot{m}_1}{\rho g_c}v_1^{\,2} - \frac{1}{g_c}\left(\frac{\dot{m}_2}{\rho}v_2^{\,2} + \frac{\dot{m}_3}{\rho}v_3^{\,2}\right) + \frac{\dot{m}_1 g}{\rho g_c}z_1 - \frac{g}{g_c}\left(\frac{\dot{m}_2}{\rho}z_2 + \frac{\dot{m}_3}{\rho}z_3\right)$$

replacing all pressure terms by their corresponding fluid column height (for example $p_1 = \frac{\rho g H_1}{g_c}$, $\Delta p_{F1-4} = \frac{\rho g \Delta H_{F1-4}}{g_c}$, etc.), and all mass flow terms by their corresponding volumetric flow rate q, $\dot{m}_1 = \rho q_1$ etc., we obtain:

$$q_1 \Delta H_P = q_1(\Delta H_{F1-4} + \Delta H_{EQ1-4}) + q_2(\Delta H_{F4-2} + \Delta H_{EQ4-2}) + q_3(\Delta H_{F4-3} + \Delta H_{EQ4-3}) -$$

$$q_1 H_1 + (q_2 H_2 + q_3 H_3) + \frac{1}{2g}(-q_1 v_1^2 + (q_2 v_2^2 + q_3 v_3^2)) - q_1 z_1 + (q_2 z_2 + q_3 z_3)$$

and since $q_1 = q_2 + q_3$, the Total Head is:

$$\Delta H_P = (\Delta H_{F1-4} + \Delta H_{EQ1-4}) + (\Delta H_{F4-2} + \Delta H_{EQ4-2}) + \frac{1}{2g}(v_2^2 - v_1^2) + (z_2 + H_2 - (z_1 + H_1)) +$$

$$\frac{q_3}{q_1}(\Delta H_{F4-3} + \Delta H_{EQ4-3}) - (\Delta H_{F4-2} + \Delta H_{EQ4-2}) + \frac{1}{2g}(v_3^2 - v_2^2) + (z_3 + H_3 - (z_2 + H_2)) \qquad [2\text{-}16]$$

For equation [2-16] to be true, the friction loss in the major branches has to be adjusted by closing the manual valves to balance the flows. The term EQ2 represents equipment on branch 2 such as a valve (see Figure 2-14). Closing the valve, reduces the flow at branch 2 but increase it at branch 3. The conditions in one branch affects the other because of the common connection point 4. The correct flow is obtained at point 3 by adjusting components in branch 2 (for example ΔH_{F4-3}, ΔH_{EQ4-3} and H_4). One would think that the main branch 2, the branch with the highest head and flow requirement, should determine the pump Total Head. So, why is it that the head requirements for branch 3 appear in equation [2-16]? All of the terms associated with branch 3 can be made to disappear after simplification.

Determine the value of H_4 using equation [2-14] and a control volume that surrounds branch 3:

$$H_4 = (\Delta H_{F4-3} + \Delta H_{EQ4-3}) + \frac{1}{2g}(v_3^2 - v_{4-3}^2) + (z_3 + H_3 - z_4)$$

or

$$(\Delta H_{F4-3} + \Delta H_{EQ4-3}) = H_4 - \frac{1}{2g}(v_3^2 - v_{4-3}^2) - (z_3 + H_3 - z_4) \qquad [2\text{-}17]$$

We can also determine the value of H_4 with equation [2-14] and a control volume surrounding branch 2:

$$H_4 = (\Delta H_{F4-2} + \Delta H_{EQ4-2}) + \frac{1}{2g}(v_2{}^2 - v_{4-2}{}^2) + (z_2 + H_2 - z_4)$$

or

$$(\Delta H_{F4-2} + \Delta H_{EQ4-2}) = H_4 - \frac{1}{2g}(v_2{}^2 - v_{4-2}{}^2) - (z_2 + H_2 - z_4) \qquad [2\text{-}18]$$

v_{4-3} and v_{4-2} are respectively the velocity at point 4 for branch 3 and branch 2, substituting equations [2-17] and [2-18] into equation [2-16] we obtain:

$$\Delta H_P = (\Delta H_{F1-4} + \Delta H_{EQ1-4}) + (\Delta H_{F4-2} + \Delta H_{EQ4-2}) + \frac{1}{2g}(v_2{}^2 - v_1{}^2) + (z_2 + H_2 - (z_1 + H_1))$$

$$+ \frac{q_3}{q_1}\left(\frac{1}{2g}(v_{4-3}{}^2 - v_{4-2}{}^2)\right) \qquad [2\text{-}19]$$

Point 4 is the point at which the flow begins to divide and the fluid particles move towards the direction of branch 2 or branch 3. All fluid particles have the same velocity at point 4 although their trajectory is about to change.

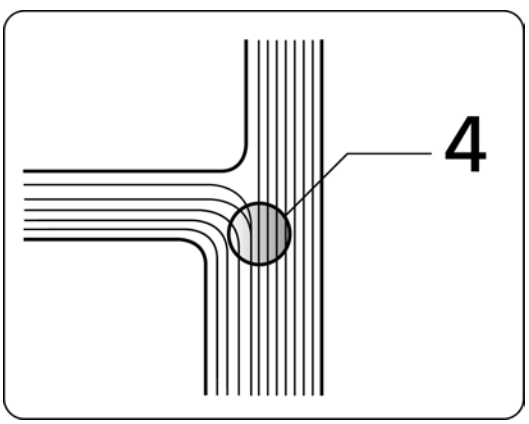

Figure 2-15 Location of H_4.

$$v_4 = v_{4-3} = v_{4-2} \qquad [2\text{-}20]$$

When we replace equation [2-20] into [2-19] then the last term in [2-19] disappears and we obtain equation [2-21] which is the equation for a single branch system. This means that the branch has no impact on the Total Head as long as branch 2 represents the path requiring the highest energy.

$$\Delta H_P = \Delta H_{F1-2} + \Delta H_{EQ1-2} + \frac{1}{2g}(v_2{}^2 - v_1{}^2) + z_2 + H_2 - (z_1 + H_1) \qquad [2\text{-}21]$$

In many cases the path requiring the highest energy is obvious. What happens if the flow in branch 3 is equal to the flow in branch 2? Alternatively, if the flow is much smaller in branch 3, but the elevation of point 3 is higher than point 2?

These situations make judging which branch will produce the higher head difficult to do at a glance. To resolve this, calculate the Total Head for the fluid traveling through either branch and compare the results. The Total Head for the pump as calculated considering the flow through branch 3 is:

$$\Delta H_P = \Delta H_{F1-3} + \Delta H_{EQ1-3} + \frac{1}{2g}(v_3{}^2 - v_1{}^2) + z_3 + H_3 - (z_1 + H_1) \quad [2\text{-}22]$$

For the system to work as intended, the highest Total Head as calculated from equations [2-21] or [2-22] will be used for sizing the pump.

2.12 GENERAL METHOD FOR DETERMINING TOTAL HEAD IN A SYSTEM WITH MULTIPLE INLETS AND OUTLETS

The energy rate balance for any system is: $\dot{Q}_E - \dot{W} = \Delta\dot{E}n + \Delta\dot{K}E + \Delta\dot{P}E$

Figure 2-16 represents a system with (n) inlets and (m) outlets.

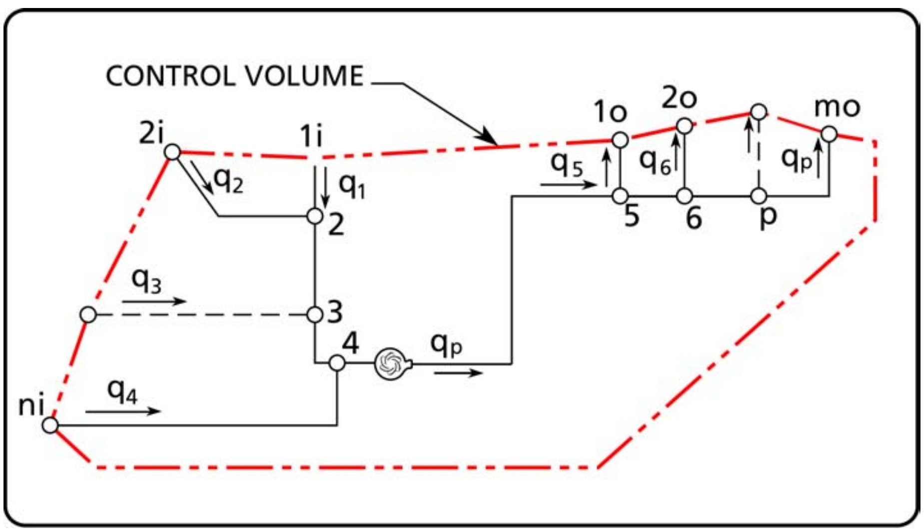

Figure 2-16 A general system with a single pump and multiple inlets and outlets.

In the system shown in Figure 2-16 there are 1 to p branches. These subscripts are used to identify the flow rates (for example q_1, q_2, ..q_p) There are li to ni inlets, which are used to identity the inlet properties (for example v_{1i}, H_{1i}, z_{1i}, v_{2i}, H_{2i}, z_{2i}, ... v_{ni}, H_{ni}, z_{ni}). Similarly, there are 1o to mo outlets that identify the outlet properties.

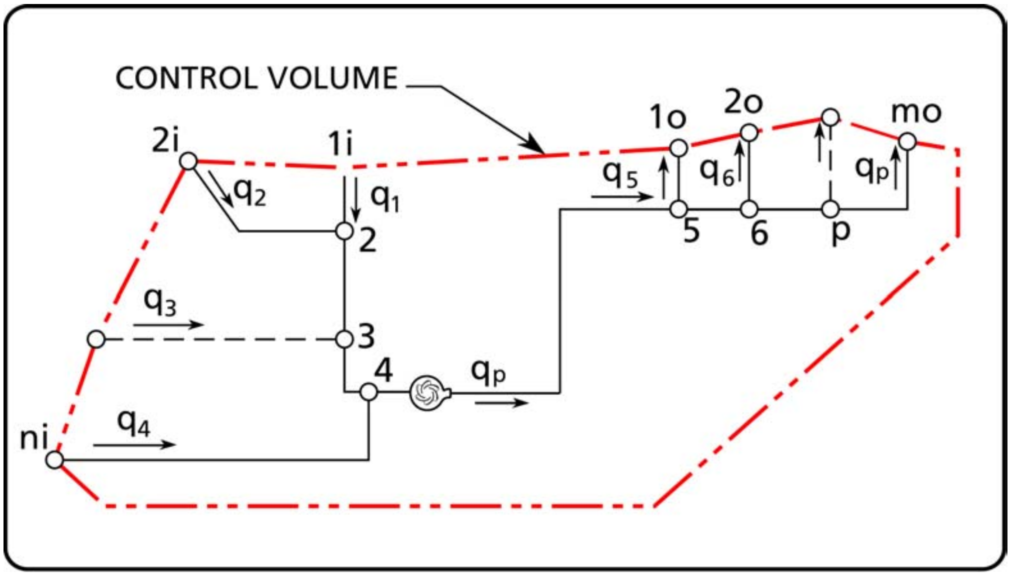

Figure 2-17 The use of the control volume with a generalized system representation.

There is one path, which for a given flow rate, will require the highest Total Head. Often, this path is obvious, but several paths may have to be checked to find the most critical.

The following equations are all based on the general principle for energy rates in a fluid medium:

$$ENERGY\ \ RATE = v\Delta F = \dot{V}\Delta P = \frac{\dot{m}}{\rho}\Delta p \propto q\Delta H$$

The friction term $\dot{Q}_E$ is:

$$q_{BR1}(\Delta H_{F1} + \Delta H_{EQ1}) + q_{BR2}(\Delta H_{F2} + \Delta H_{EQ2}) + ... + q_{BRp}\Delta H_P = \sum_{t=1}^{t=p} q_t(\Delta H_{Ft} + \Delta H_{EQt}$$

where the subscripts 1 to p stand for branch no.1, 2, etc,. up to branch no. p.

The work rate term $\dot{W}$ is:

$$q_P \Delta H_P$$

The enthalpy rare term $\Delta\dot{E}n$ is:

$$q_{1i}H_{1i} + q_{2i}H_{2i} + ... + q_{ni}H_{ni} - (q_{1o}H_{1o} + q_{20}H_{2o} + ... + q_{mo}H_{mo}) = \sum_{t=1}^{t=n} q_{ti}H_{ti} - \sum_{t=1}^{t=m} q_{to}H_{to}$$

The kinetic energy rate term $\Delta\dot{K}E$ is:

$$\frac{1}{2g}\left((q_{1i}v^2{}_{1i} + q_{2i}v^2{}_{2i} + ... + q_{ni}v^2{}_{ni} - (q_{1o}v^2{}_{1o} + q_{20}v^2{}_{2o} + ... + q_{mo}v^2{}_{mo})\right) = \frac{1}{2g}\left(\sum_{t=1}^{t=n} q_{ti}v^2{}_{ti} - \sum_{t=1}^{t=m} q_{to}v^2{}_{to})\right)$$

The potential energy rate term $\Delta\dot{P}E$ is:

$$q_{1i}z_{1i} + q_{2i}z_{2i} + ... + q_{ni}z_{ni} - (q_{1o}z_{1o} + q_{20}z_{2o} + ... + q_{mo}z_{mo}) = \sum_{t=1}^{t=n} q_{ti}z_{ti} - \sum_{t=1}^{t=m} q_{to}z_{to}$$

The Total Head for the system ΔH_P is:

$$\boxed{q_P\Delta H_P = \sum_{t=1}^{t=p} q_t(\Delta H_{Ft} + \Delta H_{EQt}) + \frac{1}{2g}\left(\sum_{t=1}^{t=n} q_{ti}v^2{}_{ti} - \sum_{t=1}^{t=m} q_{to}v^2{}_{to}\right) + \sum_{t=1}^{t=n} q_{ti}(z_{ti} + H_{ti}) - \sum_{t=1}^{t=m} q_{to}(z_{to} + H_{to})} \qquad [2\text{-}23]$$

This is a similar equation to [2-16] with the difference that there are many inlet and outlet branches. As in equation [2-16], this equation requires that the system parameters that satisfy the flow balance be known.

2.13 GENERAL METHOD FOR DETERMINING TOTAL HEAD IN A SYSTEM WITH MULTIPLE PUMPS, INLETS AND OUTLETS

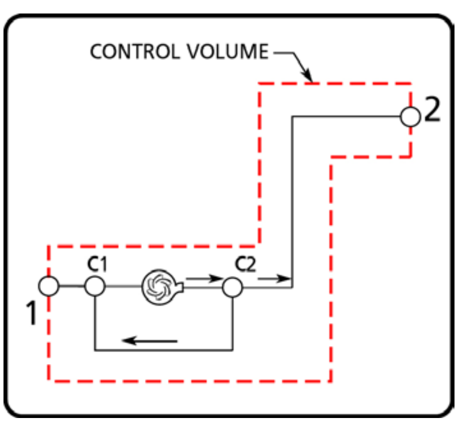

Figure 2-18 The effect of internal branches within a system.

Inlets and Outlets vs. Internal Connection Points

One of the purposes of an open system is to move material from one point to another. A more complex system may use several inlets to move material to several outlets. Only the inlets and outlets of a system have thermodynamic properties, therefore the velocity, elevation and pressure at these points are properties critical to the system. Any internal connection points with branches used to re-circulate fluid within the system do not have thermodynamic properties as regards to the system.

These branches are required for the system to function properly. For example, take a single inlet-single outlet system with a re-circulation line from the discharge to the suction of the pump (see Figure 2-18). How does this re-circulation branch affect the system if the same flow is required at point 2 with or without the re-circulation branch?

The energy rate balance is: $\dot{Q}_E - \dot{W} = \Delta\dot{E}n + \Delta\dot{K}E + \Delta\dot{P}E$

$\dot{Q}_E$ is affected in the above equation since there is more friction due to the extra branch. $\dot{W}$ is also affected since the pump will require a greater capacity to compensate for the extra fluid that must go through the branch. The effect of the extra branch is to increase the total friction of the system and therefore the work to be provided by the pump.

Multiple Pumps

The system in Figure 2-19 appears at first glance impossible to solve or at the least challenging. This is the same system as in Figure 2-17 with additional pumps and branches. Before doing any Total Head calculations, determine the flow rates for each branch. The flow through each one of the branches and pumps must be known and this requires knowledge of the purpose of each pump. Each pump will move fluid through its respective branches at its design flow rate. To determine the Total Head of pump A in Figure 2-19, in your mind disconnect all pumps and branches that are not intended to be powered by pump A and apply equation [2-23].

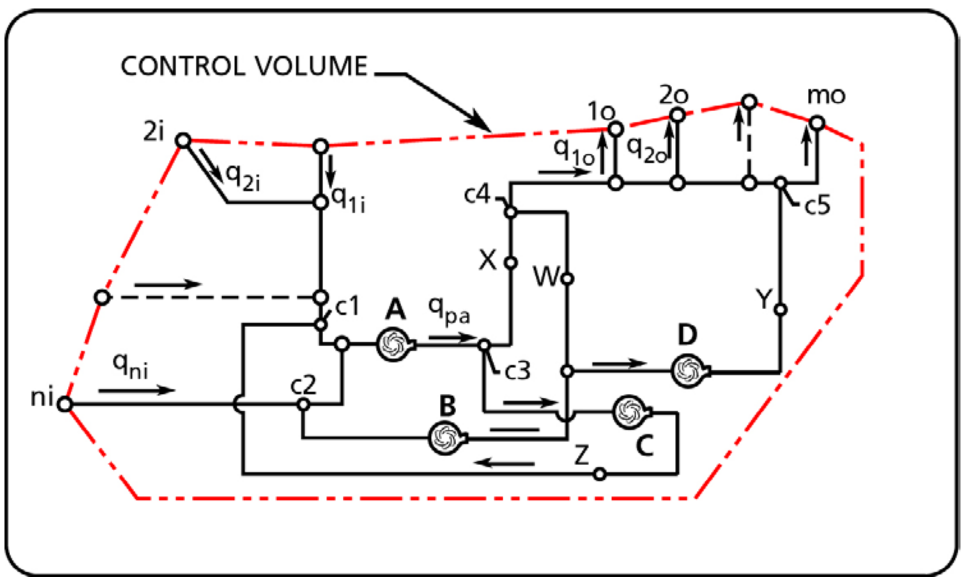

Figure 2-19 The use of the control volume in a multiple pump generalized system.

We now have a much simpler system as shown in Figure 2-20. This is a similar system to the one shown in Figure 2-17 that can be resolved with equation [2-23]. All the connecting points (c1 to c5) are points where additional flow enters or leaves some part of the system. These points are internal to the system, and not inlet or outlet points as point

1i..ni and lo...mo. The branches left behind with pump A contain all the inlets and outlets of the system. In a way, we could say that pump A is essential to the system since all the inlets and outlets are attached to pump A. The other pumps and their branches form sub-systems of the main system.

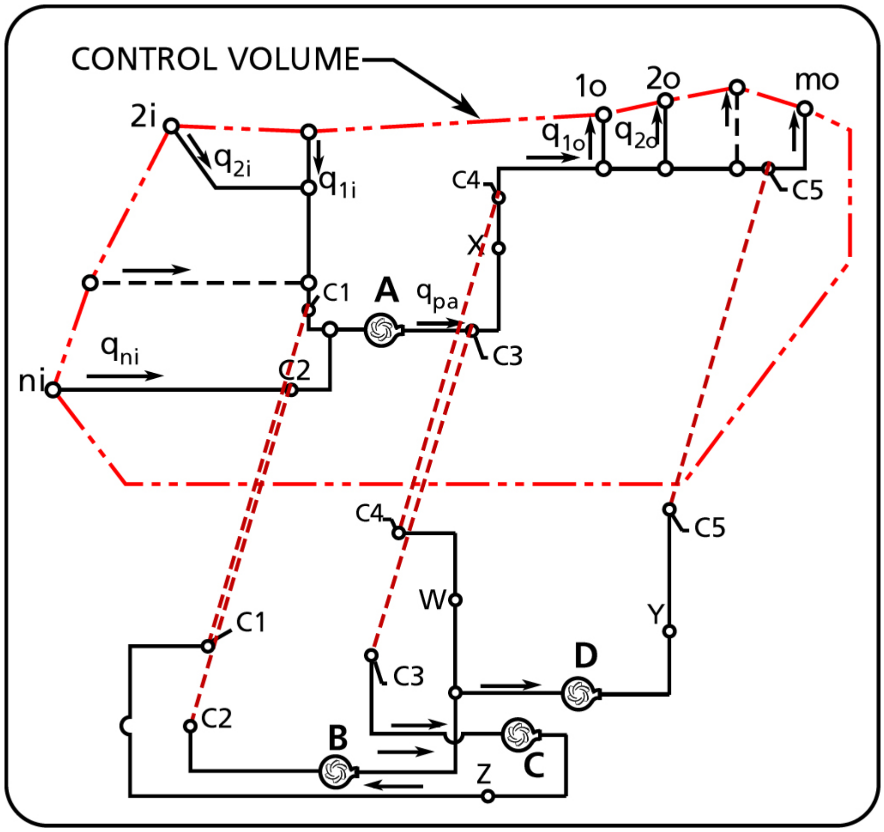

Figure 2-20 Simplifying complex systems.

To determine the Total Head of pump B in Figure 2-20, first calculate the head at points c2 and c4 (see equation [2-15] for example) and then apply equation [2-23]. The Total Heads of pump C and D are similarly determined.

2.14 GENERAL METHOD FOR DETERMINING THE PRESSURE HEAD ANYWHERE IN A SYSTEM WITH MULTIPLE PUMPS, INLETS AND OUTLETS

First, isolate each pump and its respective branches as in the previous section and calculate the Total Head of each pump. Assuming point X to be located somewhere after pump A in pump A's system, then the pressure head at point X is:

$$Q_X \Delta H_X = -Q_{PA}\Delta H_{PA} + \sum_{t=BR1}^{t=BRp} Q_t(\Delta H_{Ft} + \Delta H_{EQt}) + \frac{1}{2g}\left(\sum_{t=1}^{t=n} Q_{ti} v_{ti}^{2} - \sum_{t=1}^{t=m} Q_X v_X^{2}\right) + \sum_{t=1}^{t=n} Q_{ti}(z_{ti} + H_{ti}) - Q_X z_X \qquad [2\text{-}24]$$

if point X is before pump A then the same equation applies with $\Delta H_{PA} = 0$. To calculate the pressure head at points w, y, and z, identify their respective sub-systems and apply equation [2-13] if they are single inlet- single outlet systems or equation [2-24].

This chapter was a mouthful. The shock of dealing with thermodynamic properties is compensated by the fact that they are quite easily determined. The thermodynamic properties at the inlet of a system are the enthalpy, the kinetic and potential energies. These are respectively proportional to: pressure, velocity and elevation. The heat loss is the sum of all the losses that occur due to fluid movement through the system and the effect of equipment. Whatever is left over between the total thermodynamic energy inlet-outlet difference and the heat loss is the work required by the pump. When, we have calculated the work required of the pump or its Total Head.

A general method for determining the Total Head for a single inlet-outlet system, leads to a method for determining the pressure head anywhere within the system. So that now, when the instrumentation guy asks what is the pressure before the control valve and what is the delta p, give it to him.

A general method for calculating the Total Head in a single inlet-outlet system leads to a method for calculating the Total Head in a multiple inlet and outlet system. The heat or friction loss is calculated the same way no matter how many branches there is in a system. The difference between the heat loss and the net thermodynamic energy is of course the work required by the pump.

Once more on to the breech…

Since we came this far, might as well go all out. The next step was to find a method to solve multiple inlet and outlet systems with multiple pumps, because when pumps are added to a system, they are put there for a reason, Oh, nooooh, you say. It is always possible to convert a multiple pump system to a single pump system. The pumps and branches that are extracted can be analyzed separately without affecting the single pump system assuming that you know or calculate the pressure head, velocity and elevation of the connecting points.